Lecture 18, class activity. Collections of sets.

We will prove the general forms of DeMorgan's Law now.

1. Recall DeMorgan's Laws that we have proved before:

$$(A \cap B)^c = A^c \cup B^c, \quad (A \cup B)^c = A^c \cap B^c.$$

Prove these using a Venn diagram or (after converting to propositions) a truth table.

2. Now let $[n]$ denote the set: $[n] = \{1, 2, 3, \ldots, n\}$. Let Q_i be a sequence of sets. Prove that

$$\bigcup_{i \in [n]} Q_i = \left(\bigcup_{i \in [n-1]} Q_i \right) \cup Q_n, \qquad \bigcap_{i \in [n]} Q_i = \left(\bigcap_{i \in [n-1]} Q_i \right) \cap Q_n.$$

3. Ok, now we will prove (by induction) the general form of DeMorgan's Laws:

$$\left(\bigcup_{i \in [n]} Q_i \right)^c = \bigcap_{i \in [n]} Q_i^c, \qquad \left(\bigcap_{i \in [n]} Q_i \right)^c = \bigcup_{i \in [n]} Q_i^c.$$