

Lecture 17, class activity. Power sets.

The main goal today is to show that there cannot be a surjective function from A to $\mathcal{P}(A)$ for any A .

1. Let $A = \{1, 2, 3\}$.
 - (a) Write down a function $f: A \rightarrow \mathcal{P}(A)$ — any function you like.
 - (b) Now write down an injective function (if your last one wasn't injective).
 - (c) Is it possible to write down a surjective function from $A = \{1, 2, 3\}$ to $\mathcal{P}(A)$? Explain.

2. (**Hard!**) Ok, now we consider a general A (could be infinite!)
 - (a) Assume we have $g: A \rightarrow \mathcal{P}(A)$ that is surjective.
 - (b) Define $S = \{x \in A : x \notin g(x)\}$.
 - (c) Explain why there must be a $y \in A$ with $g(y) = S$.
 - (d) Ok, now see what happens if $y \in S$. What happens if $y \notin S$?