Name:____________________________

Lecture 14, class activity. Induction Part 2.

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1. Consider the sums of odd numbers

$$1, \quad 1+3, \quad 1+3+5, \quad 1+3+5+7, \quad 1+3+5+7+9, \ldots$$

   (a) Compute several of these and identify the pattern.

   (b) Denote S_n as the sum of the first n odd numbers, and write it in summation notation.

   (c) State and prove a theorem that gives an exact formula for S_n.

2. Identify what is wrong with the following proof:

   Theorem. All natural numbers are interesting.

   Proof. Let $I \subset \mathbb{N}$ be the set of interesting natural numbers. Let $B = \mathbb{N} \setminus I$ be the set of noninteresting, or boring, natural numbers. Assume that $B \neq \emptyset$.

   Then B has a minimal element, call it r. But note that r is the smallest natural number that is not interesting...

   ...and that's kind of interesting.

   Therefore $r \in I$, which contradicts $r \in B$.