Name:____________________________

Lecture 12, class activity. Functions Part II.

In all the problems below, we have A, B, C are sets, and

$$f\colon A \to B, \quad g\colon B \to A.$$

1. We know the function $f\colon \mathbb{Z} \to \{0, 1, 2\}$ given by

$$f(x) = r,$$

where r is the unique remainder (modulo 3) given by the Division Algorithm.

- Show that this function is surjective, but not injective.
- Show that if we restrict the domain to the set $A = \{4, 8, 9\}$, then $f\colon A \to \{0, 1, 2\}$ is bijective.
- Compute the inverse f^{-1}.
- Show that if we take the domain to be $B = \{0, 1, 2, 3, 4, 5\}$ then $f\colon B \to \{0, 1, 2\}$ is not injective.

2. We proved in class that f, g injective $\implies$ $g \circ f$ injective.

In this problem, you are going to show that the converse is (half) false.

- Show that if $g \circ f$ is injective, then f must be injective.
- Give an example to show that it is possible that $g \circ f$ is injective, but g is not injective.
  Hint. Try taking $A = C$ to be finite sets and B to be a "bigger" set.