so $y(c) = b + \int_a^c \underbrace{f(x, y(x))\,dx}$ for every c.

x is a dummy variable
rename $x \to t$
$c \to x$

equivalent to write
$$y(x) = b + \int_a^x f(t, y(t))\,dt$$

In fact: $\left\{ \begin{array}{l} \frac{dy}{dx} = f(x, y(x)) \\ y(a) = b \end{array} \right\}$ is equivalent to this equation

Consider the operator:

$$\underset{\text{function}}{y(x)} \longrightarrow \underset{\text{function}}{P[y]}(x) = b + \int_a^x f(t, y(t))\,dt$$

$y(x)$ solves the IVP if and only if $P[y](x) = y(x)$.

To produce a solution:

1) start with constant function $y_0(x) = b$
2) apply P many times $y_1 = P[y_0]$, $y_2 = P[y_1]$, $y_{n+1} = P[y_n]$
3) The functions $y_n(x)$ are <u>approximate solutions</u>

of $y(x) = \lim_{n\to\infty} y_n(x)$. In <u>good cases</u>, this will be a <u>genuine solution</u>.