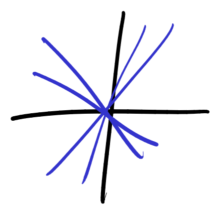
Existence and uniqueness ; Picard iteration

Suppose we have a differential equation initial value problem :
$$\begin{cases} \dfrac{dy}{dx} = f(x, y(x)) \\ y(a) = b \end{cases}$$

How do we know it has any solutions at all?
Will the solutions be unique?

Recall $\left\{ \begin{array}{l} \dfrac{dy}{dx} = \dfrac{y}{x} \\ y(0) = b \end{array} \right\}$



Has no solution if $b \neq 0$
infinitely many if $b = 0$

Or: what if equation is really wierd and we can't find solutions no matter how hard we try?

Goal: Build a solution "abstractly".
An algorithm that applies to any first order ordinary differential equation: <u>Picard iteration.</u>

Suppose y satisfies $\begin{cases} \dfrac{dy}{dx} = f(x, y(x)) \\ y(a) = b \end{cases}$

Integrate $\int_a^c dx$: $\int_a^c \frac{dy}{dx} dx = \int_a^c f(x, y(x)) dx$

$$\underset{y(c) - y(a)}{\overset{\int_a^c \frac{dy}{dx}dx}{\shortparallel}} = y(c) - b$$