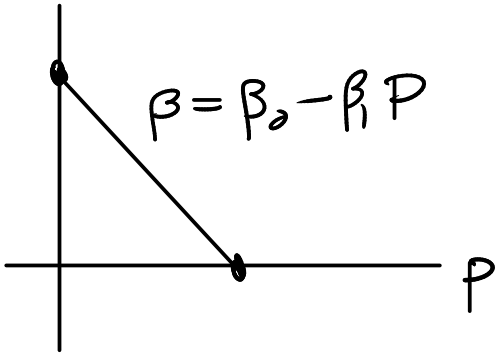
A reasonable model is that the birth rate decreases with increased population. This will be true if there are environmental factors, such as a limited food supply, which limit reproduction at high population levels.

Then $\beta = \beta(P)$ will look like



Negative slope $-\beta_1$
Hits 0 at $P = \frac{\beta_0}{\beta_1}$

Assume Death rate is constant $\delta = \delta_0$

So our population model becomes.

$$\frac{dP}{dt} = (\beta_0 - \beta_1 P - \delta_0)P = (\beta_0 - \delta_0)P - \beta_1 P^2$$

$$= \beta_1 P\left(\frac{\beta_0 - \delta_0}{\beta_1} - P\right)$$

Or $\underbrace{\frac{dP}{dt} = kP(M-P)}$ $\quad k = \beta_1$, $M = \frac{\beta_0 - \delta_0}{\beta_1}$

This is called the <u>logistic equation.</u>

Many different situations are modeled by this equation