

Population models (examples of Autonomous DEs)

Let t denote time, and let $P(t)$ denote the population of some type of organism. (bacteria, humans, deer, ...)

We introduce

β = birth rate = births per unit time per unit of population.

δ = death rate

Over an interval of time Δt , there will be

$\beta \cdot P \cdot \Delta t$ births
and

$\delta \cdot P \cdot \Delta t$ deaths

so the change in population will be

$$\Delta P \approx \beta \cdot P \cdot \Delta t - \delta \cdot P \cdot \Delta t = (\beta - \delta) P \Delta t$$

$$\frac{\Delta P}{\Delta t} \approx (\beta - \delta) P$$

in the limit as $\Delta t \rightarrow 0$, we obtain $\frac{dP}{dt} = (\beta - \delta) P$