$$e^{kt_1}T(t_1) - e^{k\cdot 0}T(0) = \int_0^{t_1} ke^{kt}A(t)dt$$

But $e^{k\cdot 0}T(0) = T(0) = T_0$ which is known.

So $$e^{kt_1}T(t_1) = T_0 + \int_0^{t_1} ke^{kt}A(t)dt$$

$$T(t_1) = T_0 e^{-kt_1} + e^{-kt_1}\int_0^{t_1} ke^{kt}A(t)dt$$

So we have a precise formula interms of the initial condition T_0 and the definite integral of an expression involving $A(t)$.

A general idea is that using definite integrals makes the management of constants of integration more precise.