Let's see what the method says:

Standard form: $\frac{dT}{dt} + kT = kA(t)$

Integrating factor $e^{\int k\,dt} = e^{kt}$

Multiply $e^{kt}\frac{dT}{dt} + ke^{kt}T = ke^{kt}A(t)$

$$\frac{d}{dt}\left(e^{kt}T\right) = ke^{kt}A(t)$$

$$e^{kt}T = \int ke^{kt}A(t)\,dt + C$$

$$T = e^{-kt}\int ke^{kt}A(t)\,dt + Ce^{-kt}$$

What about initial condition $T(0) = T_0$?
It's hard to say because we are writing things in terms of indefinite integrals.

We can be more precise if we use definite integrals

Start from $\frac{d}{dt}\left(e^{kt}T\right) = ke^{kt}A(t)$

Now integrate $\int_0^{t_1} - \,dt$ $\quad 0 =$ initial time, $t_1 =$ some later time

$$\int_0^{t_1}\frac{d}{dt}\left(e^{kt}T\right)dt = \int_0^{t_1} ke^{kt}A(t)\,dt$$