Let's assume we are only interested in $\boxed{x>0}$

$P(x) = -\frac{1}{x} \Rightarrow$ integrating factor $e^{\int -\frac{1}{x}dx} = e^{-\ln x} = x^{-1}$

Note that we could write $\int -\frac{1}{x}dx = -\ln x$ because we only care about $x>0$.

Multiply by x^{-1} $\quad x^{-1}\frac{dy}{dx} - x^{-2}y = x^{-1}x^2 = x$

$$\frac{d}{dx}\left(x^{-1}y\right) = x$$

$$x^{-1}y = \frac{1}{2}x^2 + C$$

$$\boxed{y = \frac{1}{2}x^3 + Cx \text{ is the general solution for } x>0.}$$

We can use this method to get formulas that are useful in numerical calculation, even if they are not totally explicit:

Newton's Law of cooling with <u>variable ambient temperature</u>.

Consider $\frac{dT}{dt} = -k(T-A)$ where $A = A(t)$ is a function of time.

One may think we are controlling the temperature in the room where an object with temperature $T(t)$ sits.

Let's also impose an initial condition $T(0) = T_0$