Step 2: multiply the <u>entire equation</u> by the Integrating factor

$$e^{3x}\frac{dy}{dx} + 3e^{3x}y = e^{3x}\cdot 2x\cdot e^{-3x} = 2x$$

Step 3: Verify that left hand side is a derivative

$$\frac{d}{dx}\left(e^{3x}y\right) = e^{3x}\frac{dy}{dx} + 3e^{3x}y = 2x$$

Step 4: Integrate

$$e^{3x}y = \int 2x\,dx = x^2 + C$$

Step 5: Solve for y

$$y = x^2e^{-3x} + Ce^{-3x}$$

<u>If initial condition is given</u>: Step 6 solve for constant C

Say $y(0) = 1$. $\quad 1 = 0^2e^{-3\cdot 0} + Ce^{-3\cdot 0} = C$

So $y(x) = x^2e^{-3x} + e^{-3x}$ is particular solution

<u>Another example</u> $\quad x\frac{dy}{dx} - y = x^3$

STANDARD FORM: $\quad \frac{dy}{dx} - \frac{1}{x}y = x^2$

The slope field is undefined at $x=0$, so we only study solutions for $x>0$ OR $x<0$.