

Going back to the equation, with $f(x) = e^{\int P(x) dx}$

$$e^{\int P(x) dx} \frac{dy}{dx} + e^{\int P(x) dx} P(x) y = e^{\int P(x) dx} Q(x)$$

$$\frac{d}{dx} \left(e^{\int P(x) dx} y \right) = e^{\int P(x) dx} Q(x)$$

Can finally integrate!

$$e^{\int P(x) dx} y = \int \left(e^{\int P(x) dx} Q(x) \right) dx + C$$

Now the constant does matter.

Solve for y :

$$y = e^{-\int P(x) dx} \int \left(e^{\int P(x) dx} Q(x) \right) dx + C e^{-\int P(x) dx}$$

Let's see it in action

$$\frac{dy}{dx} + 3y = 2x e^{-3x}$$

$$P(x) = 3$$

$$Q(x) = 2x e^{-3x}$$

Step 1 find integrating factor
So we can use e^{3x}

$$e^{\int P(x) dx}$$

$$\int 3 dx = 3x$$