To solve, we want to be able to integrate LHS, meaning we need to <u>recognize it as a derivative</u>.

The trick is to multiply the <u>whole equation</u> by a function $\rho(x)$ first.

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\underbrace{\rho(x)\frac{dy}{dx} + \rho(x)P(x)y} = \rho(x)\,Q(x)$$

This is reminiscent of the product rule

$$\frac{d}{dx}\left(\rho(x)y\right) = \rho(x)\frac{dy}{dx} + \frac{d\rho}{dx}y$$

What if we pick $\rho(x)$ so that $\frac{d\rho}{dx} = \rho(x)\,P(x)$?

This equation is separable:

$$\frac{1}{\rho}\frac{d\rho}{dx} = P(x) \longrightarrow \int \frac{1}{\rho}\,d\rho = \int P(x)\,dx$$

so $\ln|\rho| = \int P(x)\,dx$

$$\rho = Ce^{\int P(x)\,dx}$$

We only need one such ρ, so we just take

$$\rho(x) = e^{\int P(x)\,dx}$$

[This happens to be a situation where the constant of integration doesn't matter.]