

# First Order Linear equations (integrating factors)

This is an important and rather general class of equations that can be solved in terms of integrals

Definition a first-order linear equation is one that can be written as

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$P(x)$  and  $Q(x)$  are the coefficient functions.

Call Linear because  $\frac{dy}{dx}$  and  $y$  appear to first power ONLY.

$$\text{Eg } \frac{dy}{dx} = xy \iff \frac{dy}{dx} - xy = 0 \quad \begin{array}{l} P(x) = -x \\ Q(x) = 0 \end{array}$$

$$\frac{dT}{dt} = -k(T-A) \iff \frac{dT}{dt} + kT = kA \quad \begin{array}{l} P(t) = k \\ Q(t) = kA \end{array}$$

$$x \frac{dy}{dx} - y = x^3 \iff \frac{dy}{dx} - \frac{1}{x}y = x^2 \quad \begin{array}{l} P(x) = -\frac{1}{x} \\ Q(x) = x^2 \end{array}$$

\* We always want the equation in STANDARD FORM  
$$\frac{dy}{dx} + P(x)y = Q(x)$$