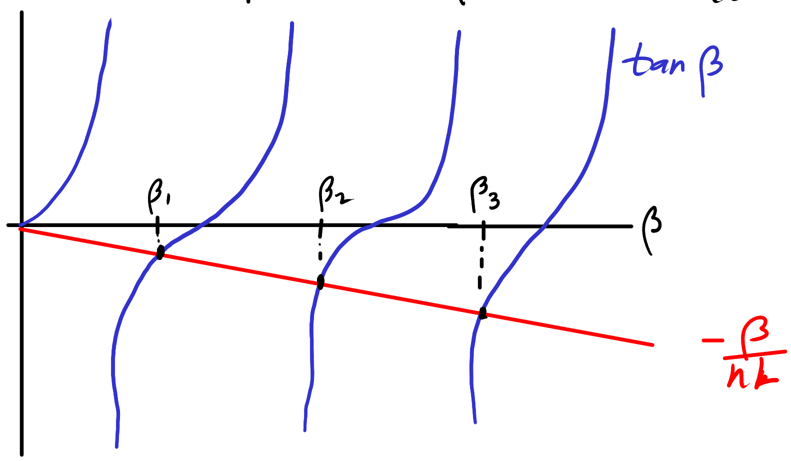
To see solutions, plot $\tan\beta$ and $\frac{\beta}{hL}$



Let β_n denote the nth positive root of the equation

$$\tan\beta = -\frac{\beta}{hL}$$

Thus $\lambda_n = \alpha_n^2 = (\beta_n/L)^2$ are the eigenvalues and

$y_n(x) = \sin\alpha_n x = \sin\frac{\beta_n x}{L}$ are the eigenfunctions.

__Theorem 2__ Suppose that a Sturm–Liouville problem satisfies the hypotheses of Theorem 1(a). Then the eigenfunctions $y_n(x)$ satisfy

$$\int_a^b y_n(x)\,y_m(x)\,r(x)\,dx = 0 \quad \text{if } n \neq m.$$

[In words, eigenfunctions for distinct eigenvalues are orthogonal with respect to the weighted inner product $\langle f,g\rangle = \int_a^b f(x)g(x)r(x)\,dx$]

__Illustration:__ Let $y_n = \sin\frac{\beta_n x}{L}$, $0<x<L$ be the eigenfunctions

Then if $n \neq m$, $\int_0^L \sin\left(\frac{\beta_n x}{L}\right)\sin\frac{\beta_m x}{L}\,dx = 0$

This is interesting, because we know this even though we don't know what β_n and β_m are exactly.