Illustration: $y'' + \lambda y = 0 \quad (0 < x < L)$

$$y(0) = 0, \quad h y(L) + y'(L) = 0 \quad (h > 0).$$

This satisfies the hypotheses of the theorem: $p(x) = 1 > 0$, $r(x) = 1 > 0$ so it satisfies clause (a). Also $q(x) = 0 \geq 0$ and $\alpha_1 = 1 \geq 0$, $\alpha_2 = 0 \geq 0$ $\beta_1 = h \geq 0$, $\beta_2 = 1 \geq 0$ so it satisfies clause (b).
Thus we know from the theorem that there are no negative eigenvalues.

$\underline{\lambda = 0}$: $y = Ax + B$, $y(0) = 0 \Rightarrow B = 0$.

$$hy(L) + y'(L) = hAL + A = (hL+1)A \Rightarrow A = 0$$

So $\lambda = 0$ is not an eigenvalue

$\underline{\lambda > 0}$: Write $\lambda = \alpha^2$, so $y(x) = A\cos\alpha x + B\sin\alpha x$

$$y(0) = 0 \Rightarrow A = 0 \quad \text{so} \quad y(x) = B\sin\alpha x$$

Now
$$\begin{aligned} 0 = hy(L) + y'(L) &= hB\sin\alpha L + \alpha B\cos\alpha L \\ &= B(h\sin\alpha L + \alpha\cos\alpha L) \end{aligned}$$

B will be forced to be zero unless

$$h\sin\alpha L + \alpha\cos\alpha L = 0$$

$$h\sin\alpha L = -\alpha\cos\alpha L$$

$$h\tan\alpha L = -\alpha$$

$$\tan\alpha L = -\frac{\alpha}{h}$$

Set $\beta = \alpha L$, the equation becomes

$$\tan\beta = -\frac{\beta}{hL}$$

This is a transcendental equation that cannot be solved analytically (as far as I know)