Endpoint conditions: Domain $[a,b]$

At $x=a$: $\alpha_1 y(a) - \alpha_2 y'(a) = 0$

At $x=b$: $\beta_1 y(b) + \beta_2 y(b) = 0$

[Assume $\alpha_1^2+\alpha_2^2>0$, $\beta_1^2+\beta_2^2>0$]

Sturm-Liouville problem : To determine those pairs $(\lambda, y(x))$ satisfying

(1) $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] - q(x)y + \lambda r(x) y = 0$

(2) $\alpha_1 y(a) - \alpha_2 y'(a) = 0$

(3) $\beta_1 y(b) - \beta_2 y(b) = 0$

Obviously $y(x)=0$ and λ = anything is a solution.
The values of λ for which a solution $y(x)$ not constant $=0$ exists are called eigenvalues. The corresponding functions $y(x)$ are called eigenfunctions.

We consider this problem so that we can state a general theorem.

Theorem 1 (a) Suppose that $p(x), p'(x), q(x), r(x)$ are continuous on $[a,b]$ And $p(x)>0$ and $r(x)>0$ for all points in $[a,b]$.
Then the eigenvalues form an increasing sequence

$$\lambda_1 < \lambda_2 < \cdots < \lambda_n < \cdots, \quad \lim_{n\to\infty} \lambda_n = \infty.$$

The eigenfunction $y_n(x)$ associated to λ_n is unique up to multiplication by a constant.

(b) Suppose additionally that $q(x) \geq 0$ on $[a,b]$, and that the coefficients $\alpha_1, \alpha_2, \beta_1, \beta_2$ are nonnegative.
Then all eigenvalues are nonnegative $\lambda_n \geq 0$.