! So $u_n(x,y) = \sinh\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n\pi x}{a}\right)$ satisfies $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$,

and $u_n(0,y)=0$, $u_n(a,y)=0$, $u_n(x,0)=0$!

General solution of Laplace eqn and the three homogeneous boundary conditions:

$$u(x,t) = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n\pi x}{a}\right)$$

2 NOTA BENE: It makes a difference which side has the nonhomogeneous boundary condition !!!

Continuing: We still need to satisfy $u(x,b)=f(x)$

$$u(x,b) = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi b}{a}\right)\sin\left(\frac{n\pi x}{a}\right) \stackrel{?}{=} f(x)$$

Again this is a sine series for $f(x)$, with $L=a$:

$$c_n \sinh\left(\frac{n\pi b}{a}\right) = b_n = \frac{2}{a}\int_0^a f(x)\sin\frac{n\pi x}{a}\,dx$$

So $$c_n = \left[\sinh\left(\frac{n\pi b}{a}\right)\right]^{-1}\frac{2}{a}\int_0^a f(x)\sin\frac{n\pi x}{a}\,dx$$

<u>Example</u>: if $f(x) = x$, then

$$b_n = \frac{2}{a}\int_0^a x\sin\frac{n\pi x}{a}\,dx = \frac{2a(-1)^{n+1}}{n\pi} \Rightarrow f(x) = x = \sum_{n=1}^{\infty}\frac{2a(-1)^{n+1}}{n\pi}\sin\frac{n\pi x}{a}$$