These boundery conditions are very much nonhomogeneous,
So we can't apply separation of variables directly.

On the other hand for homogeneous boundary conditions
the only solution is $u=0$.

$$\begin{array}{ccc} & u=0 & \\ u=0 & \boxed{\Delta u=0} & u=0 \\ & u=0 & \end{array}$$

We want to reduce to the case where all but one of the four sides carries a homogeneous boundary condition.

If u_L solves
$$\begin{array}{ccc} & u_L=0 & \\ u_L=g_1(y) & \boxed{\Delta u_L=0} & u_L=0 \\ & u_L=0 & \end{array}$$
(left)

If u_R solves
$$\begin{array}{ccc} & u_R=0 & \\ u_R=0 & \boxed{\Delta u_R=0} & u_R=g_2(y) \\ & u_R=0 & \end{array}$$
(right)

If u_T solves
$$\begin{array}{ccc} & u_T=f_2(x) & \\ u_T=0 & \boxed{\Delta u_T=0} & u_T=0 \\ & u_T=0 & \end{array}$$
(top)

If u_B solves
$$\begin{array}{ccc} & u_B=0 & \\ u_B=0 & \boxed{\Delta u_B=0} & u_B=0 \\ & u_B=f_1(x) & \end{array}$$
(bottom)

Then $u=u_L+u_R+u_T+u_B$ solves
$$\begin{array}{ccc} & u=f_2(x) & \\ u=g_1(y) & \boxed{\Delta u=0} & u=g_2(y) \\ & u=f_1(x) & \end{array}$$
!