Laplace equation I

We consider functions of two variables $u(x,y)$, where now both x and y are thought of as spatial variables.

The two-dimensional Laplace equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Solutions of Laplace equation are also called harmonic functions.

Similarly, the three-dimensional Laplace equation is for a function $u(x,y,z)$,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Note that

$$\operatorname{div}(\operatorname{grad} u) = \nabla \cdot (\nabla u) = \nabla \cdot \left\langle \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z} \right\rangle$$
$$= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Also written $\nabla^2 u$ or Δu, called the Laplacian of u.
The operator

$$\Delta = \nabla^2 = \operatorname{div}\operatorname{grad} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ is

called the Laplacian or Laplace operator.

In 2d: $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. In 1d: $\Delta = \frac{d^2}{dx^2}$

This is one of the most important operators in mathematics.