Wave Equation II

Let's recall the problem: Vibrating string

Domain: $0 \le x \le L$ $\quad$ L = length of string

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2} & \text{Wave eqn.} \\ u(0,t) = 0 \\ u(L,t) = 0 & \left.\begin{array}{l}\text{boundary conditions} \\ \text{Ends of string fixed}\end{array}\right\} \\ u(x,0) = f(x) & \text{initial position} \\ \dfrac{\partial u}{\partial t}(x,0) = g(x) & \text{initial velocity.} \end{cases}$$

This problem may be solved by separation of variables. It is very much analogous to what we did to solve the heat equation. The steps are:

1) Write $u(x,t) = T(t)X(x)$. Figure out the ordinary DEs that T and X must satisfy in order for u to satisfy the partial DE.

2) Translate Boundary conditions into endpoint conditions on X. You should get an eigenvalue problem for X.

3) Solve the eigenvalue problem: get eigenvalues λ_n eigenfunctions X_n.