Wave equation: For waves in water, air, vibrating string, ...

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$u(x,t)$ = water level, air pressure, displacement of vibrating medium.

We will solve the vibrating string using separation of variables.

But first:

To break up the monotony a bit, let's look at the wave eqn with out boundary conditions, i.e., x ranges over the whole real line.

There is an amazingly elegant solution found by D'Alembert:

Let $F(z)$ and $G(z)$ be functions (assume F'' and G'' exist)

Then

$$u(x,t) = F(x-ct) + G(x+ct)$$

Satisfies $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$