The solution of $\dfrac{d^2 X}{dx^2} + \lambda X = 0$ depend on whether $\lambda<0, \lambda=0, \lambda>0$

$\lambda > 0$: $X(x) = A\cos\sqrt{\lambda}\,x + B\sin\sqrt{\lambda}\,x$

$\lambda = 0$: $X(x) = A + Bx$

$\lambda < 0$: $X(x) = A e^{\sqrt{-\lambda}\,x} + B e^{-\sqrt{-\lambda}\,x}$

Now, $X(x)\,T(t)$ is a solution of the heat equation!

so: if $\lambda > 0$: $u = e^{-k\lambda t}\left(A\cos\sqrt{\lambda}\,x + B\sin\sqrt{\lambda}\,x\right)$

satisfies $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$

if $\lambda = 0$: $u = e^{-k0t}(A+Bx) = A+Bx$

satisfies it: infact, for these solutions

$\dfrac{\partial u}{\partial t} = 0$ "steady-state solution"

if $\lambda < 0$, let $a^2 = -\lambda$: Then

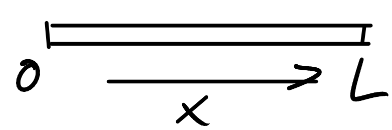
$u = e^{ka^2 t}\left(Ae^{ax} + Be^{-ax}\right)$ solves it.

these solutions go to $\pm\infty$ as $t \to \infty$, so they don't have much physical meaning.

This is really great: we have lots of solutions to play with. There is even a free parameter λ that we can vary.

We have too many! Use boundary conditions to cut them down

Let's consider the Rod of length L



* Endpoints held fixed at temperature 0:

$$\left.\begin{aligned} u(0,t) &= 0 \\ u(L,t) &= 0 \end{aligned}\right\} \text{ Boundary conditions}$$