Heat Equation II

Still trying to solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$. Last time, we sought factorizable solutions of the form $u(x,t) = X(x)T(t)$

$$\frac{dT}{dt}(t)\,X(x) = k\frac{d^2X}{dx^2}(x)\,T(t)$$

$$\frac{1}{kT(t)}\frac{dT}{dt}(t) = \frac{1}{X(x)}\frac{d^2X}{dx^2}(x)$$

Recall: LHS doesn't depend on x, RHS doesn't depend on t, so both are constant. We call the constant $-\lambda$.

$$\frac{1}{kT}\frac{dT}{dt} = -\lambda = \frac{1}{X}\frac{d^2X}{dx^2}$$

i.e. $\frac{d^2X}{dx^2} = -\lambda X$ and $\frac{dT}{dt} = -k\lambda T$

i.e. $\left\{\begin{array}{l} \frac{d^2X}{dx^2} + \lambda X = 0 \\ \\ \frac{dT}{dt} = -k\lambda T \end{array}\right\}$ for some value of λ.

These are ordinary differential equations and we know how to solve them!

$$\frac{dT}{dt} = -k\lambda T \implies T(t) = Ce^{-k\lambda t} \quad (C \text{ constant})$$