Thus if $\lambda = \left(\frac{n\pi}{7}\right)^2$ we have a nontrivial solution
$y = C_1 \cos\left(\frac{n\pi}{7} x\right) \Rightarrow \lambda$ is an eigenvalue

Otherwise, the constant C_1 must be zero, and then $y=0$ is the only solution. $\Rightarrow \lambda$ is not an eigenvalue

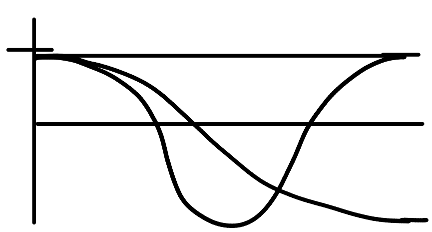
What about $\lambda = 0$? $y'' = 0 \Rightarrow y = C_1 + C_2 x \Rightarrow y' = C_2$
$y'(0) = 0 \Rightarrow C_2 = 0 \Rightarrow y = C_1 \Rightarrow y' = 0$
so $y'(7) = 0$ automatically!
$\lambda = 0$ is an eigenvalue, the eigenfunction is $y = C$ constant

Summary $\left\{ \begin{array}{l} y'' + \lambda y = 0 \\ y'(0) = 0 \\ y'(7) = 0 \end{array} \right\}$ the eigenvalues are
$\lambda = 0, \left(\frac{\pi}{7}\right)^2, \left(\frac{2\pi}{7}\right)^2, \left(\frac{3\pi}{7}\right)^2, \ldots$
$\lambda_n = \left(\frac{n\pi}{7}\right)^2 \quad n = 0, 1, 2, 3, \ldots$

Associated to the eigenvalue $\lambda_n = \left(\frac{n\pi}{7}\right)^2$ is the basic eigenfunction

$$y_n(x) = \cos \frac{n\pi x}{7} \qquad (y_0(x) = \cos 0 = 1)$$

The other eigenfunctions are $C y_n(x) = C \cos \frac{n\pi x}{7}$ for C constant.



horizontal tangents at both ends.

Other variations

$$\left\{ \begin{array}{l} y'' + \lambda y = 0 \\ y(0) = 0 \\ y'(1) = 0 \end{array} \right\}$$