To summarize

$$\left\{\begin{array}{l} y''+\lambda y=0 \\ y(0)=0 \\ y(1)=0 \end{array}\right\}$$

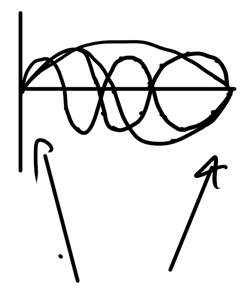
The positive eigenvalues are $\lambda = \pi^2, (2\pi)^2, (3\pi)^2, \dots$

That is $\lambda = (n\pi)^2 \quad n=1,2,3,\dots$

Associated to the eigenvalue $\lambda_n = (n\pi)^2$, we have the basic eigenfunction:

$$y_n(x) = \sin n\pi x$$

the other eigenfunctions are $Cy_n(x) = C\sin n\pi x$ where C is any constant.



zero at endpoints

What about $\lambda \le 0$?

Is $\lambda = 0$ an eigenvalue?

$$\left\{\begin{array}{l} y''=0 \\ y(0)=0 \\ y(1)=0 \end{array}\right\}$$

$y''=0 \Rightarrow y = c_1 + c_2 x$

$y(0)=0 \Rightarrow 0 = c_1 \Rightarrow y = c_2 x$

$y(1)=0 \Rightarrow 0 = c_2 \Rightarrow y \equiv 0 \Rightarrow y$ is trivial

So 0 is <u>not an eigenvalue</u>.

What about $\lambda < 0$?

$$y''+\lambda y = 0$$

$r^2+\lambda = 0 \qquad r = \pm\sqrt{-\lambda}$ this is real

$$y = c_1 e^{\sqrt{\lambda}x} + c_2 e^{-\sqrt{\lambda}x}$$

$y(0)=0 \Rightarrow c_1 + c_2 = 0$

$y(1)=0 \Rightarrow c_1 e^{\sqrt{\lambda}} + c_2 e^{-\sqrt{\lambda}} = 0 \Rightarrow c_1 e^{2\sqrt{\lambda}} + c_2 = 0$

$\Rightarrow c_1 e^{2\sqrt{\lambda}} - c_1 = 0 \Rightarrow c_1(e^{2\sqrt{\lambda}} - 1) = 0 \Rightarrow c_1 = 0$

$\Rightarrow c_2 = -c_1 = 0$ so $y \equiv 0 \Rightarrow y$ is trivial

So $\lambda < 0$ is <u>not an eigenvalue</u>.