Eigenvalues and Eigenfunctions

Last time: End point value problem

$$\begin{cases} y'' + \lambda y = 0 \\ y(0) = 0 \\ y(1) = 0 \end{cases}$$

If $\lambda > 0$ there are two possibilities
- $\lambda = (n\pi)^2$ where n is an integer
Then we have
$$y(x) = C \sin n\pi x \quad \text{as a solution}$$
for any constant C

- $\lambda \neq (n\pi)^2$, the only solution is $y(x) \equiv 0$.
  (y is constant equal to 0)

By convention, we declare the function $y(x) \equiv 0$
which is constantly equal to zero to be
"uninteresting" or "trivial" or "nugatory".

All other functions are called
"interesting" or "nontrivial".

So our problem only has interesting/nontrivial solutions
if $\lambda = (n\pi)^2$ for some integer n.
The values of λ for which the problem has
interesting/nontrivial solutions are called eigenvalues.

The interesting/nontrivial solutions themselves
are called eigenfunctions or modes.

The set of eigenvalues is called the spectrum of the
problem.