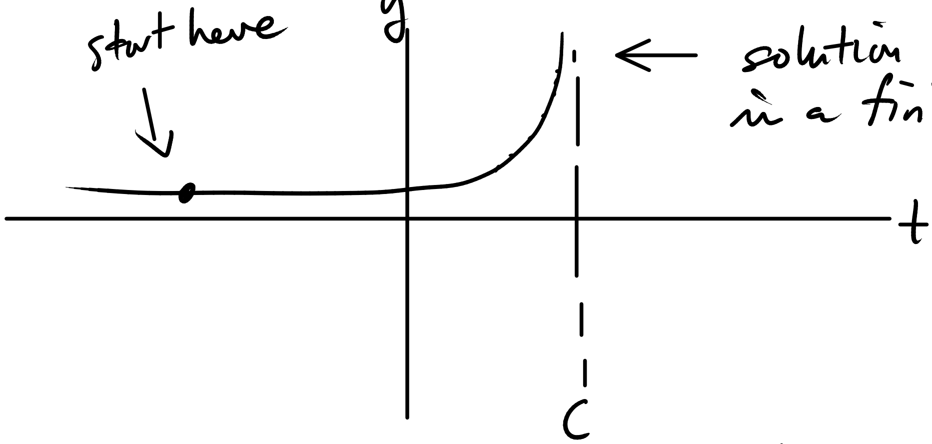
2. <u>Eventual failure of existence</u> $\frac{dy}{dt} = y^2$

Solutions are $y(t) = \frac{1}{C-t}$ or $y(t) = 0$



← solution blows up (goes to ∞) in a finite amount of time

The solution cannot be extended to a continuous function for $t \geq C$.

3. <u>Failure of uniqueness</u> $\frac{dy}{dx} = \frac{2y}{x} \Rightarrow y(x) = Cx^2$

All values of C will satisfy initial condition $y(0)=0$!

There are some natural conditions that rule out 1. and 3. (but Not 2.!)

Consider $\frac{dy}{dt} = f(t,y)$.

Informally: if $f(t,y)$ and $\frac{\partial f}{\partial y}(t,y)$ are <u>continuous</u>

then a solution of $\left\{ \begin{array}{l} \frac{dy}{dt} = f(t,y) \\ y(a) = b \end{array} \right\}$ exists for times close to $t=a$, and the solution is unique.