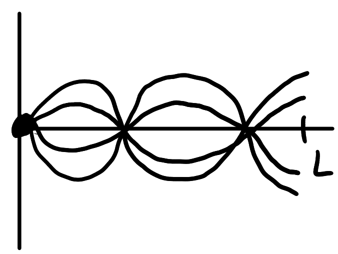
Thus, there is no existence and uniqueness theorem for Endpoint problems.
Instead, the question of whether solutions exist and how many becomes the focus of our study.

Consider the general class of problems

$$\begin{cases} y'' + \lambda y = 0 \\ y(0) = 0 \\ y(L) = 0 \end{cases}$$

Here λ and L are parameters we can vary to get different problems.

One way to think about what this problem is asking
Start with initial condition $y(0)=0$. Chose any initial velocity $y'(0)$ that you want, let $y(x)$ evolve according to the differential equation $y''+\lambda y=0$, and see where it ends up at $x=L$



← same initial position, same diff eq, different initial velocities, different $y(L)$ values.

So let's solve as far as possible for as many λ's as possible

$$\begin{cases} y'' + \lambda y = 0 \\ y(0) = 0 \\ y(L) = 0 \end{cases}$$

<u>Case</u> $\lambda = 0$

$$y''=0 \Rightarrow y = c_1 + c_2 x$$
$$y(0)=0 \Rightarrow c_1 = 0 \Rightarrow y = c_2 x$$
$$y(L)=0 \Rightarrow c_2 L = 0 \Rightarrow c_2 = 0 \Rightarrow y = 0$$

So $y \doteq 0$ is the unique solution.