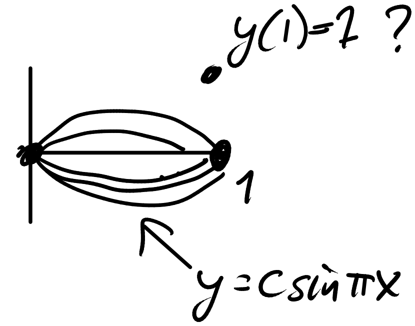
$\therefore \quad y = C_2 \sin x$

$y(\frac{\pi}{2}) = 7 \Rightarrow C_2 \sin\frac{\pi}{2} = 7 \Rightarrow C_2 = 7$

So $y = 7\sin x$ is the only solution.



__Ex 2:__ $\begin{cases} y'' + \pi^2 y = 0 \\ y(0) = 0 \\ y(1) = 1 \end{cases}$

$y'' + \pi^2 y \Rightarrow y = C_1 \cos \pi x + C_2 \sin \pi x$

$y(0) = 0 \Rightarrow C_1 \cos 0 + C_2 \sin 0 = 0$

$\Rightarrow C_1 = 0$

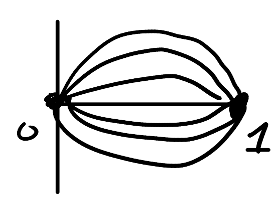
So $y = C_2 \sin \pi x$. Then $y(1) = 1 \Rightarrow C_2 \sin \pi = 1$

$\Rightarrow C_2 \cdot 0 = 1$

$\Rightarrow 0 = 1$

This is absurd, so this endpoint problem has __no solution__.

But how about $\begin{cases} y'' + \pi^2 y = 0 \\ y(0) = 0 \\ y(1) = 0 \end{cases}$



$y = C_1 \cos \pi x + C_2 \sin \pi x$

$C_1 \cos 0 + C_2 \sin 0 = 0$

$C_1 = 0$

So $y = C_2 \sin \pi x \qquad y(1) = 0 \Rightarrow C_2 \sin \pi = 0$

This is always true since $\sin \pi = 0$

So this problem has __infinitely many solutions__:

$$\boxed{y = C \sin \pi x \quad \text{for any constant } c}$$