Endpoint value problems and eigenvalues

Previously we have studied Initial value problems eg.

$$\begin{cases} ay'' + by' + cy = f(x) & \leftarrow \text{Differential equation} \\ \left.\begin{array}{l} y(0) = b_0 \\ y'(0) = b_1 \end{array}\right\} & \text{initial conditions} \end{cases}$$

Under reasonable assumptions, this problem always has a <u>unique solution</u>.

Contrast to this the endpoint value problem

$$\begin{cases} ay'' + by' + cy = f(x) & \leftarrow \text{Differential equation} \\ \left.\begin{array}{l} y(0) = b_0 \\ y(L) = b_1 \end{array}\right\} & \text{endpoint conditions.} \end{cases}$$

We are trying to specify the value of the solution at Different values of x.

Eg. What are the solutions of

$$\begin{cases} y'' + y = 0 \\ y(0) = 0 \\ y(\frac{\pi}{2}) = 7 \end{cases}$$

$y'' + y = 0 \implies y = c_1 \cos x + c_2 \sin x$ for some constant c_1 and c_2

$y(0) = 0 \Rightarrow c_1 \cos 0 + c_2 \sin 0 = 0 \longrightarrow c_1 = 0$

$c_1 1 + c_2 \cdot 0 = 0$