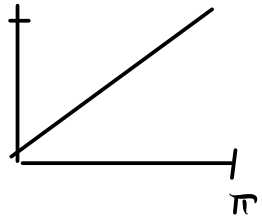
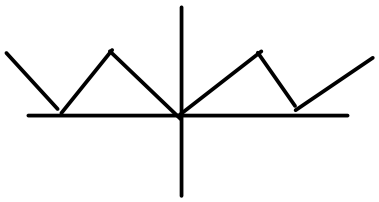
Example, $f(t)$



$f(t) = t$ on $0 < t < \pi$

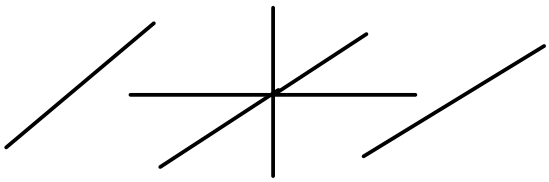
$f_{even}(t)$ triangle!



$f(t) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd}} \frac{1}{n^2} \cos nt$

$(0 < t < \pi)$

$f_{odd}(t)$ sawtooth!



$f(t) = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nt$

$(0 < t < \pi)$

So the equations

$$t = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos t + \frac{1}{3^2}\cos 3t + \frac{1}{5^2}\cos 5t + \cdots\right)$$

and

$$t = 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \cdots\right)$$

are <u>both</u> valid for $0 < t < \pi$!

Application of Fourier series: Solving Forced oscillator

$mx'' + kx = F(t)$ for <u>any</u> periodic driving force $F(t)$.

Suppose $F(t)$ is periodic with period $2L$

Expand $F(t)$ as a Fourier series

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)$$