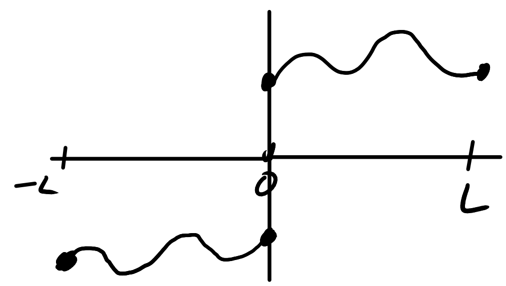
Then we can forget about values of $t$ outside of $[0, L]$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} \quad (\text{Domain } 0 \le t \le L)$$

Thus we have represented the original function as a cosine series

2. Odd extension of period $2L$

Define $f_{odd}(t)$ for $-L < t < 0$ by $f_{odd}(t) = -f(-t)$



Then repeat with period $2L$

Since extended function is odd, Fourier series is a sine series

$$b_n = \frac{2}{L}\int_0^L f(t) \sin\frac{n\pi t}{L}\,dt$$

$$f_{odd}(t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{L} \quad (\text{Domain } -\infty < t < \infty)$$

Considering only $t$ in $[0, L]$:

$$f(t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{L} \quad (\text{Domain } 0 \le t \le L)$$

The same function $f(t)$ on $[0, L]$ has both a cosine series and a sine series.