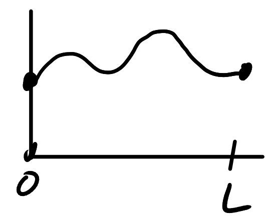
Fourier Sine and Cosine series; applications

[Review even and odd functions, see last page of previous lecture]

We will sometimes apply Fourier series methods with functions that are <u>not</u> period, but rather, defined on an interval. The idea is to extend to a period (even or odd) function, and then take the Fourier series expansion.

Let $f(t)$ be defined on the interval $[0, L]$. It's not periodic or anything like that.

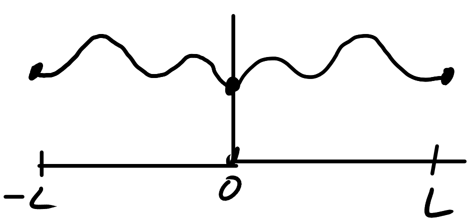


To get a Fourier Series, we <u>extend</u> $f(t)$ to a periodic function on the whole line.

There are several choices of how to do this.

1. Even extension of period $2L$

Define $f_{even}(t)$ for $-L < t < 0$ by $\underset{\text{in } [-L,0]}{f_{even}(t)} = \underset{\text{in } [0,L]}{f(-t)}$



Then make it periodic with period $2L$.
Since the extended function is even, it's Fourier series is a cosine series

$$a_0 = \frac{2}{L}\int_0^L f(t)\,dt \qquad a_n = \frac{2}{L}\int_0^L f(t)\cos\frac{n\pi t}{L}\,dt$$

$$f_{Even}(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} \quad (\text{Domain } -\infty < t < \infty)$$