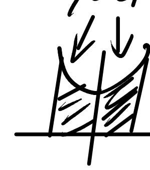
Recall $F(t)$ is <u>even</u> if $F(-t) = F(t)$

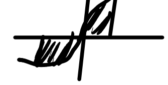
<u>odd</u> if $F(-t) = -F(t)$

equal areas



If $F(t)$ is <u>even</u> then $\int_{-a}^{a} F(t)\,dt = 2\int_{0}^{a} F(t)\,dt$

If $F(t)$ is <u>odd</u> then $\int_{-a}^{a} F(t)\,dt = 0$

 areas cancel

<u>If $f(t)$ is even</u>, then $f(t)\cos\left(\frac{n\pi t}{L}\right)$ is even, and $f(t)\sin\left(\frac{n\pi t}{L}\right)$ is odd.

look $f(-t)\cos\left(\frac{n\pi(-t)}{L}\right) = f(t)\cos\frac{n\pi t}{L}$

$f(-t)\sin\left(\frac{n\pi(-t)}{L}\right) = f(t)\left(-\sin\frac{n\pi t}{L}\right) = -f(t)\sin\frac{n\pi t}{L}$

So $a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt = \frac{2}{L}\int_{0}^{L} f(t)\,dt$

$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}\,dt = \frac{2}{L}\int_{0}^{L} f(t)\cos\frac{n\pi t}{L}\,dt$

$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}\,dt = 0$

<u>If $f(t)$ is odd</u>; then $f(t)\cos\frac{n\pi t}{L}$ is odd and $f(t)\sin\frac{n\pi t}{L}$ is even

So $a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt = 0$

$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}\,dt = 0$

$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}\,dt = \frac{2}{L}\int_{0}^{L} f(t)\sin\frac{n\pi t}{L}\,dt$