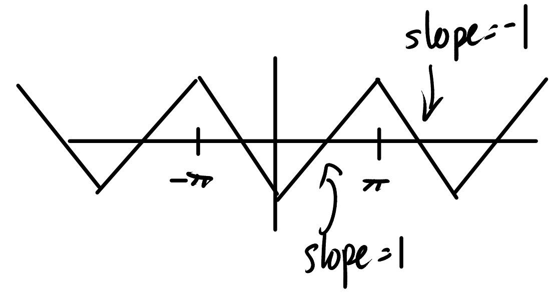
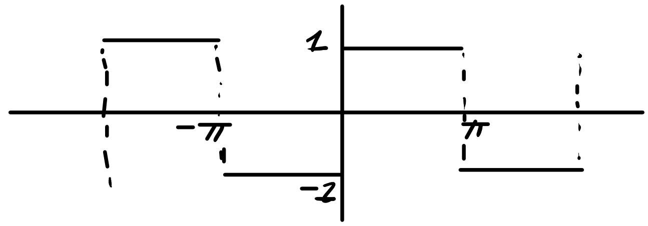
Example Triangle wave $f(t) =$



$f'(t) =$



Square wave!

We know Fourier series of $f'(t)$

$$f'(t) = \frac{4}{\pi} \sum_{n \text{ odd}} \frac{1}{n} \sin nt$$

What is the Fourier series of $f(t)$?

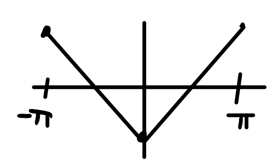
$$f(t) - f(0) = \int_0^t f'(s)\,ds = \int_0^t \frac{4}{\pi} \sum_{n\text{ odd}} \frac{1}{n} \sin ns\, ds$$

$$= \frac{4}{\pi} \sum_{n\text{ odd}} \frac{1}{n} \int_0^t \sin ns\,ds = \frac{4}{\pi} \sum_{n\text{ odd}} \frac{1}{n} \left[\frac{-1}{n} \cos ns \right]_{s=0}^{s=t}$$

$$= \frac{4}{\pi} \sum_{n\text{ odd}} \frac{-1}{n^2} (\cos nt - 1) = \frac{4}{\pi} \sum_{n\text{ odd}} \frac{1}{n^2} - \frac{4}{\pi} \sum_{n\text{ odd}} \frac{1}{n^2} \cos nt$$

So
$$f(t) = \underbrace{\left(f(0) + \frac{4}{\pi} \sum_{n\text{ odd}} \frac{1}{n^2} \right)}_{\text{just a constant.}} - \frac{4}{\pi} \sum_{n\text{ odd}} \frac{1}{n^2} \cos nt$$

Suppose $f(t)$ is symmetrical about the t-axis



Then $a_0 = 0$, and $f(0) = -\frac{\pi}{2}$.

This implies $\displaystyle\sum_{n\text{ odd}} \frac{1}{n^2} = \frac{\pi^2}{8}$!