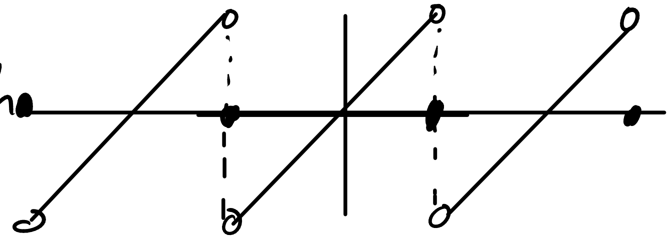
For sawtooth 

Differentiation of Fourier Series.

Suppose $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)$

If we differentiate term by term, we expect

$$f'(t) \overset{?}{=} 0 + \sum_{n=1}^{\infty}\left(-\frac{n\pi}{L} a_n \sin\frac{n\pi t}{L} + \frac{n\pi}{L} b_n \cos\frac{n\pi t}{L}\right)$$

This is <u>not always valid</u> (see HW)

It is valid as long as
- $f(t)$ is continuous and
- $f'(t)$ is piecewise smooth.

Integration works better:
Suppose $f(t)$ is piecewise continuous, and it has Fourier series

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)$$

* It is possible that the Fourier series of a merely piecewise continuous function does not converge at all. *

Even so, the equation

$$\int_0^t f(s)\,ds = \frac{a_0 t}{2} + \sum_{n=1}^{\infty}\left[\frac{L}{n\pi} a_n \sin\frac{n\pi t}{L} - \frac{L}{n\pi} b_n\left(\cos\frac{n\pi t}{L} - 1\right)\right]$$

is valid (in particular, the sum is convergent)