

Convergence of Fourier Series

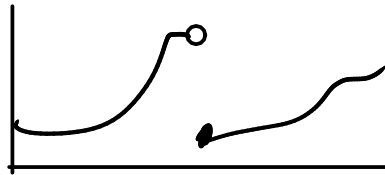
A function $f(t)$ is called smooth if

- $f(t)$ is continuous
- $f'(t)$ exist for all t
- $f'(t)$ is continuous

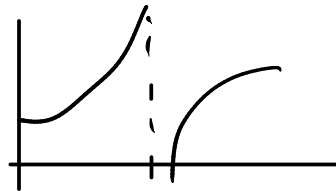
Thm If $f(t)$ is a smooth periodic function, the Fourier series of $f(t)$ converges to $f(t)$ for every t .

A function is piecewise smooth if it is smooth except for (isolated) "jump discontinuities"

Jump discontinuity



Vertical asymptote is not a jump discontinuity.



Thm If $f(t)$ is a piecewise smooth periodic function, the Fourier series of f converges to

- $f(t)$ if f is continuous at t
- $\frac{1}{2} \left[\lim_{s \rightarrow t^-} f(s) + \lim_{s \rightarrow t^+} f(s) \right]$ if t is a jump discontinuity.

Eg. Squarewave Fourier series converges to

