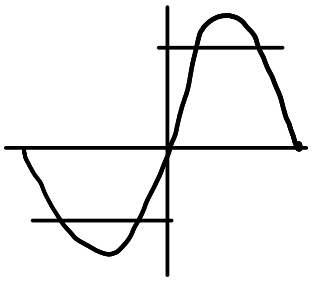
Convergence: Do the partial sums of the Fourier Series converge to the original function?

If the function is "smooth" meaning $f(t)$ is continuous, $f'(t)$ exists everywhere, and $f'(t)$ is continuous, then the answer is <u>yes</u>.

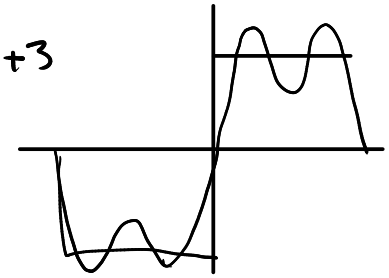
If $f(t)$ is discontinuous, the answer is <u>not necessarily</u>.

Partial sums of Fourier series for square wave

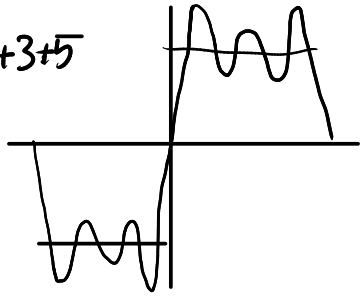
1



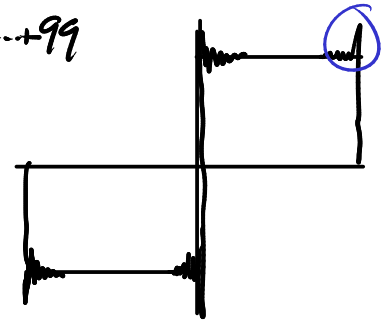
1+3



1+3+5



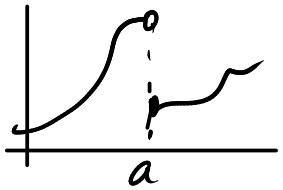
1+3+...+99



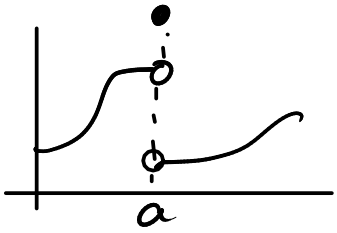
"Gibbs Phenomenon"

Basic issue: sequence of continuous functions can't converge (uniformly) to a discontinuous function!

A function is said to have a <u>jump discontinuity</u> if both one-sided limits exist, but are not equal.



or



$$\lim_{t \to a^-} f(t) \neq \lim_{t \to a^+} f(t)$$