$$a_n = \left[\frac{1}{n\pi} t \sin n\pi t + \left(\frac{1}{n\pi}\right)^2 \cos n\pi t \right]_{-1}^{1}$$

$$= \frac{1}{n\pi} \sin n\pi + \frac{1}{n\pi} \sin(-n\pi) + \left(\frac{1}{n\pi}\right)^2 \cos n\pi - \left(\frac{1}{n\pi}\right)^2 \cos(-n\pi)$$

$$= 0 + 0 + \left(\frac{1}{n\pi}\right)^2 - \left(\frac{1}{n\pi}\right)^2 = 0$$

So $a_0 = 0$ and $a_n = 0$.

We could have seen this because $f(t)$, $f(t) \cos n\pi t$ are odd functions.

- If $F(t)$ is odd: $F(-t) = -F(t)$ Then $\int_{-a}^{a} F(t)\,dt = 0$

$$b_n = \frac{1}{1} \int_{-1}^{1} f(t) \sin n\pi t\,dt = \int_{-1}^{1} t \sin n\pi t\,dt$$

Parts: $\int t \sin n\pi t\,dt = -\frac{1}{n\pi} t \cos n\pi t - \int -\frac{1}{n\pi} \cos n\pi t\,dt$

$$\boxed{\begin{array}{ll} u = t & dv = \sin n\pi t\,dt \\ du = dt & v = -\frac{1}{n\pi} \cos n\pi t \end{array}}$$

$$= -\frac{1}{n\pi} t \cos n\pi t + \left(\frac{1}{n\pi}\right)^2 \sin n\pi t + C$$

$$b_n = \left[-\frac{1}{n\pi} t \cos n\pi t + \left(\frac{1}{n\pi}\right)^2 \sin n\pi t \right]_{-1}^{1}$$

$$= -\frac{1}{n\pi} \cos n\pi + \frac{1}{n\pi}(-1) \cos(-n\pi) + \left(\frac{1}{n\pi}\right)^2 \sin n\pi - \left(\frac{1}{n\pi}\right)^2 \sin(-n\pi)$$