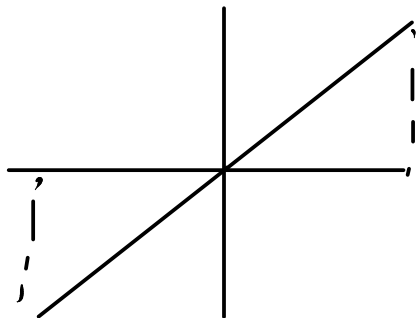
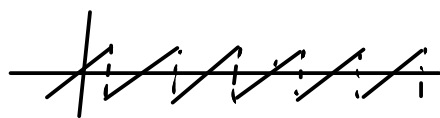


## Fourier Series; Pointwise convergence.

Another example of Fourier Series: Sawtooth wave



$$f(t) = \begin{cases} t & \text{if } -1 \leq t < 1, \\ \text{repeats periodically with} \\ \text{period } 2 \quad (L=1). \end{cases}$$



What is  $f(27.2)$ ?

$$f(27.2) = f(25.2) = f(23.2) = \dots = f(1.2) = f(-.8) = -.8$$

In general, for a function with period  $2L$ :  $f(t+2Lk) = f(t)$

Compute Fourier coefficients

$$a_0 = \frac{1}{1} \int_{-1}^1 f(t) dt = \int_{-1}^1 t dt = \left[ \frac{1}{2} t^2 \right]_{-1}^1 = \frac{1}{2} (1 - 1) = 0$$

$$a_n = \frac{1}{1} \int_{-1}^1 f(t) \cos \frac{n\pi t}{1} dt = \int_{-1}^1 t \cos n\pi t dt$$

Parts  $u = t \quad dv = \cos n\pi t dt$   
 $du = dt \quad v = \frac{1}{n\pi} \sin n\pi t$

$$\begin{aligned} \int t \cos n\pi t dt &= \frac{1}{n\pi} t \sin n\pi t - \int \frac{1}{n\pi} \sin n\pi t dt \\ &= \frac{1}{n\pi} t \sin n\pi t + \left( \frac{1}{n\pi} \right)^2 \cos n\pi t + C \end{aligned}$$