By orthogonality, all the terms go away except

$$\int_{-L}^{L} a_n \cos\frac{n\pi t}{L}\cos\frac{m\pi t}{L}\,dt \quad \text{when} \quad m=n$$

$$= a_m L$$

So $\int_{-L}^{L} f(t)\cos\frac{m\pi t}{L}\,dt$ so $a_m = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{m\pi t}{L}\,dt$

Similar argument shows $b_m = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{m\pi t}{L}\,dt$

<u>Definition</u> The <u>Fourier coefficients</u> of $f(t)$ are

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}\,dt \qquad n=1,2,\ldots$$

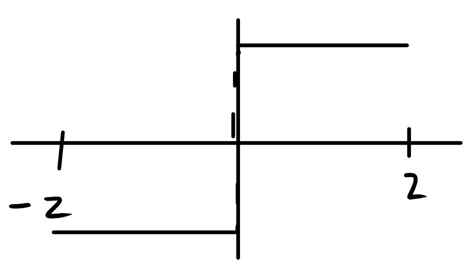
$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}\,dt \qquad n=1,2,\ldots$$

The <u>Fourier Series</u> of $f(t)$ is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi t}{L} + b_n\sin\frac{n\pi t}{L}\right)$$

<u>Examples</u>: square-wave with period $4=2L$, $L=2$.

$$f(t) = \begin{cases} 1 & 0 \le t < 2 \\ -1 & -2 \le t < 0 \end{cases}$$



Repeats periodically