To figure out constant term, just integrate from $-L$ to L

$$\int_{-L}^{L} f(t)\,dt = \int_{-L}^{L}\left(\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)dt$$

look: $$\int_{-L}^{L} \cos\frac{n\pi t}{L}\,dt = \left[\frac{L}{\pi n}\sin\frac{n\pi t}{L}\right]_{-L}^{L} = \frac{L}{\pi n}\left[\sin n\pi - \sin -n\pi\right]$$
$$= \frac{L}{\pi n}\cdot 0 = 0$$

$$\int_{-L}^{L} \sin\frac{n\pi t}{L}\,dt = \left[\frac{-L}{\pi n}\cos\frac{n\pi t}{L}\right]_{-L}^{L} = -\frac{L}{\pi n}\left[\cos n\pi - \cos(-n\pi)\right]$$
$$= \frac{-L}{\pi n}\cdot 0 = 0$$

So all of the integral goes away. except the $\frac{a_0}{2}$ term

$$\int_{-L}^{L} f(t)\,dt = \int_{-L}^{L}\frac{a_0}{2}\,dt = \frac{a_0}{2}\int_{-L}^{L} dt = \frac{a_0}{2}2L = a_0 L$$

$$\int_{-L}^{L} f(t)\,dt = a_0 L \implies a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt$$

So we found a_0 in terms of $f(t)$!

To find a_m: multiply by $\cos\frac{m\pi t}{L}$ and integrate:
use orthogonality

$$\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}\,dt = \int_{-L}^{L}\left(\frac{a_0}{2} + \sum a_n\cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)\cos\frac{m\pi t}{L}\,dt$$

$$= \int_{-L}^{L} \frac{a_0}{2}\cos\frac{m\pi t}{L} + \sum \quad a_n\cos\frac{n\pi t}{L}\cos\frac{m\pi t}{L} + b_n \sin\frac{n\pi t}{L}\cos\frac{m\pi t}{L}\Big)\,dt$$