Fourier Coefficients:

Recall orthogonality relations

Fundamental period 2π

$\sin nt$, $\cos nt$, $n = 1, 2, 3, \ldots$

$$\int_{-\pi}^{\pi} \cos mt \cos nt \, dt = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

$$\int_{-\pi}^{\pi} \sin mt \sin nt \, dt = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

$$\int_{-\pi}^{\pi} \sin mt \cos nt \, dt = 0$$

Fundamental period $2L$

$\sin \frac{n\pi t}{L}$, $\cos \frac{n\pi t}{L}$, $n = 1, 2, 3, \ldots$

$$\int_{-L}^{L} \cos \frac{m\pi t}{L} \cos \frac{n\pi t}{L} \, dt = \begin{cases} 0 & \text{if } m \neq n \\ L & \text{if } m = n \end{cases}$$

$$\int_{-L}^{L} \sin \frac{m\pi t}{L} \sin \frac{n\pi t}{L} \, dt = \begin{cases} 0 & \text{if } m \neq n \\ L & \text{if } m = n \end{cases}$$

$$\int_{-\pi}^{\pi} \sin \frac{m\pi t}{L} \cos \frac{n\pi t}{L} \, dt = 0$$

Now, suppose we are trying to write a function $f(t)$ (which we assume is periodic with period $2L$) as a sum of sines and cosines with fundamental period $2L$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$