These identities are not extremely difficult to prove, but let's set that aside and understand what is going on.

The set of functions on an interval $[a,b]$ has a version of dot product, called <u>inner product</u>

Let $f(t)$ and $g(t)$ be two functions defined on $[a,b]$
Their <u>inner product</u> is

$$f \bullet g = \langle f, g \rangle = \int_a^b f(t) g(t)\, dt$$

$$\left\{ \text{Compare:} \quad \begin{array}{l} \vec{u} = (u_1, \dots, u_n) \\ \vec{v} = (v_1, \dots, u_n) \end{array} \qquad \vec{u} \cdot \vec{v} = \langle \vec{u}, \vec{v} \rangle = \sum_{i=1}^{n} u_i v_i \right\}$$

Two functions are called <u>orthogonal</u> on $[a,b]$ if $f \bullet g = 0$

That is, f, g orthogonal $\iff \int_a^b f(t) g(t)\, dt = 0$

So we are saying

$$\langle \cos mt, \cos nt \rangle = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

$$\langle \sin mt, \sin nt \rangle = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

$$\langle \cos mt, \sin nt \rangle = 0 \quad \text{always.}$$

Where the fundamental period is 2π.