The orthogonality of sines and cosines at different frequencies.

Consider the sine and cosine functions

$\sin t$	angular freq $= 1$	$\sin 3t$	etc
$\cos t$	freq $= 1/2\pi$	$\cos 3t$	
	Period $= 2\pi$		
$\sin 2t$	angular freq $= 2$	$\sin nt$	angular freq $= n$
$\cos 2t$	freq $= 1/\pi$	$\cos nt$	freq $= n/2\pi$
	Period $= \pi$		Period $= 2\pi/n$

Q: Take the product of two such functions, and integrate over one fundamental period of length 2π

Consider two integers m, n

$$\int_{-\pi}^{\pi} \cos mt \cos nt \, dt = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

$$\int_{-\pi}^{\pi} \sin mt \sin nt \, dt = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

$$\int_{-\pi}^{\pi} \sin mt \cos nt \, dt = 0 \quad \text{for all } m \text{ and } n$$

We say that $\sin nt$ and $\cos nt$ form a family of orthogonal trigonometric functions with fundamental period 2π