Starting Fourier series

Today: Orthogonality and Fourier coefficients

Motivation: Fourier series are useful for any situation where there is periodic behavior.

We try to write $f(t)$ as a series (= infinite sum) of sines and cosines:

$$f(t) = c + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

Then we can operate on $f(t)$ term-by-term.

Some places where this is useful:

Forced oscillation:

$$m \frac{d^2x}{dt^2} + kx = f(t)$$

Can get a solution by adding solutions for

$$m \frac{d^2x}{dt^2} + kx = a_n \cos \frac{n\pi t}{L} \qquad m \frac{d^2x}{dt^2} + kx = b_n \sin \frac{n\pi t}{L}$$

Solving partial differential equations of physics.

Heat equation: $u(x,t)$: $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ k = thermal diffusion constant

Wave equation $u(x,t)$: $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial t^2}$ c = speed of wave

Schrödinger equation (free particle) $\Psi(x,t)$ $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2}$ $\hbar$ = Planck's const.