**Beats**: Let's go back to the undamped case.

$mx'' + kx = F_0 \cos \omega t$ Assume no resonance $\omega \neq \omega_0 = \sqrt{\frac{k}{m}}$

Particular solution $= P \cos \omega t$ where $P = \frac{F_0}{(k - m\omega^2)}$

General solution of homogeneous equation

$$= C \cos(\omega_0 t - \alpha)$$

So the general solution of the forced system is

$$x(t) = C \cos(\omega_0 t - \alpha) + P \cos \omega t$$

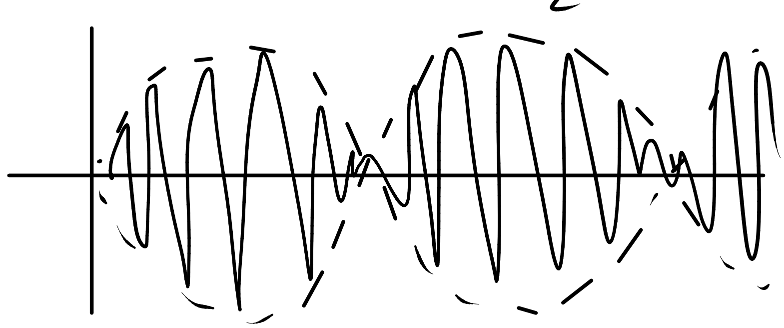
Now we use a rare trig identity "Sum-to-product"

$$\cos \theta + \cos \varphi = 2 \cos\left(\frac{\theta + \varphi}{2}\right) \cos\left(\frac{\theta - \varphi}{2}\right)$$

For simplicity consider $C(\cos \omega_0 t + \cos \omega t)$

$$\cos \omega_0 t + \cos \omega t = 2 \cos\left(\frac{\omega_0 + \omega}{2} t\right) \cos\left(\frac{\omega_0 - \omega}{2} t\right)$$

If $\omega \approx \omega_0$ then $\frac{\omega_0 + \omega}{2} \approx \omega_0$ $\frac{\omega_0 - \omega}{2}$ is very small.



**Beats**