When does $C = \dfrac{F_0}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}}$ have a maximum?

Need $\sqrt{(k-m\omega^2)^2+(c\omega)^2}$ has a minimum

Need $(k-m\omega^2)^2+(c\omega)^2$ has a minimum

$$0 = \frac{d}{d\omega}\left[(k-m\omega^2)^2+(c\omega)^2\right] = 2(k-m\omega^2)(-2m\omega)+2c^2\omega$$

$\omega=0$ is a solution, we want another one, so cancel ω

$$0=-4m(k-m\omega^2)+2c^2$$

$$0=-2m(k-m\omega^2)+c^2$$

$$(k-m\omega^2)=\frac{c^2}{2m}$$

$$k-\frac{c^2}{2m}=m\omega^2$$

$$\frac{k}{m}-\frac{c^2}{2m^2}=\omega^2$$

$$\boxed{\begin{array}{l}\omega_0=\sqrt{\frac{k}{m}}\\ \gamma=\frac{c}{2m}\end{array}}$$

$$\omega=\sqrt{\frac{k}{m}-\frac{c^2}{2m^2}}$$

$$=\sqrt{\omega_0^2-2\gamma^2}$$

This frequency may not be real, but if it is, the plot looks like