

When does  $C = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}$  have a maximum?

Need  $\sqrt{(k-m\omega^2)^2 + (c\omega)^2}$  has a minimum

Need  $(k-m\omega^2)^2 + (c\omega)^2$  has a minimum

$$0 = \frac{d}{d\omega} [(k-m\omega^2)^2 + (c\omega)^2] = 2(k-m\omega^2)(-2m\omega) + 2c^2\omega$$

$\omega = 0$  is a solution, we want another one, so cancel  $\omega$

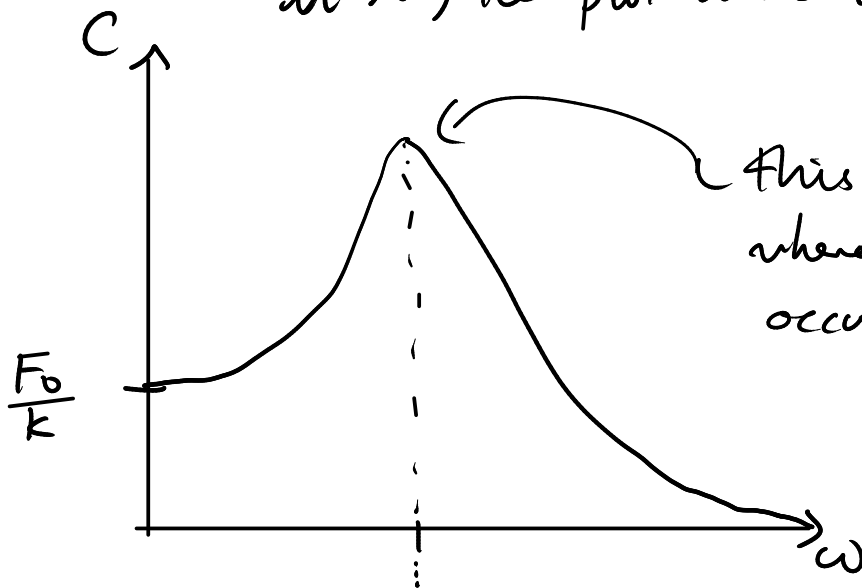
$$0 = -4m(k-m\omega^2) + 2c^2 \quad \rightarrow \quad k - \frac{c^2}{2m} = m\omega^2$$

$$0 = -2m(k-m\omega^2) + c^2$$
$$(k-m\omega^2) = \frac{c^2}{2m} \quad \rightarrow \quad \frac{k}{m} - \frac{c^2}{2m^2} = \omega^2$$

$$\omega = \sqrt{\frac{k}{m} - \frac{c^2}{2m^2}}$$
$$= \sqrt{\omega_0^2 - 2\gamma^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$
$$\gamma = \frac{c}{2m}$$

This frequency may not be real, but if it is, the plot looks like



This is the frequency where "practical resonance" occurs.